Date: 23 Feb 2021

VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

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Permutations vs. Combinations

- Both are ways to count the possibilities
- Arrangements
- The difference between them is whether order matters or not

Combinations

- **Definition :** The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations.
- **Number of combinations of n different things taken r at a time. (denoted by nCr C(n,r), Cn,r)**
- $nCr = n!/r!\,(n-r)!$

Pg 5.16 , Ex 2

- **Example 2:** Let S be the collection of eight points in the plane with no three points on the straight line. Find the number of triangles that have points of S as vertices.
- **Solution:** Every choice of three points out of S determines a unique triangle. The order of the points selected is unimportant as whatever be the order, we will get the same triangle. Hence, the desired number is the number of combinations of eight things taken three at a time. Therefore, we get
- $^8C_3 = 8!/3!5! = 8 \times 7 \times 6/3 \times 2 \times 1 = 56$ choices.

Pg 5.16 Ex 3

- **Example 3:** A committee is to be formed of 3 persons out of 12. Find the number of ways of forming such a committee.
- **Solution:** We want to find out the number of combinations of 12 things taken 3 at a time and this is given by
- 12 C3 = 12!/3!(12 – 3)! [by the definition of nCr]
- = 12!/3!9! = 12×11×10×9!/3!9! = 12×11×10/3×2 = 220

Example 4

- **Example 4:** A committee of 7 members is to be chosen from 6 Chartered Accountants, 4 Economists and 5 Cost Accountants. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?
- **Solution:** The various methods of selecting the persons from the various groups are shown below:

Analysis

	Committee of 7 members		
	C.A.s	Economists	Cost Accountants
Method 1	3	2	2
Method 2	4	2	1
Method 3	4	1	2
Method 4	5	1	1
Method 5	3	3	1
Method 6	3	1	3

- Number of ways of choosing the committee members by
- Method 1 = 6C3×4C2×5C2 =
- =20×6×10=1,200
- Method 2 = 6C4×4C2×5C1 =
- = 15×6×5 = 450
- Method 3 = 6C4×4C1×5C2 = = 15×4×10 = 600.
- Method 4 = 6C5×4C1×5C1 = 6×4×5 = 120.
- Method 5 = 6C3×4C3×5C1 == 20×4×5 = 400
- Method 6 = 6C3×4C1×5C3 == 20×4×10 = 800.
- Therefore, total number of ways = 1,200 + 450 + 600 + 120 + 400 + 800 = 3,570

- **Example 5:** A person has 12 friends of whom 8 are relatives. In how many ways can he invite 7 guests such that 5 of them are relatives?
- **Solution:** Of the 12 friends, 8 are relatives and the remaining 4 are not relatives. He has to invite 5 relatives and 2 friends as his guests. 5 relatives can be chosen out of 8 in 8C5 ways; 2 friends can be chosen out of 4 in 4C2 ways.
- Hence, by the fundamental principle, the number of ways in which he can invite 7 guests such that 5 of them are relatives and 2 are friends.
- = 8C5 × 4C2
- = 8´7´6
- = 336.

- **Example 9:** A box contains 7 red, 6 white and 4 blue balls. How many selections of three balls can be made so that (a) all three are red (b) none is red (c) one is of each colour?
- **Solution:** (a) All three balls will be of red colour if they are taken out of 7 red balls and this can be done in $7C3 = 7! / 3!(7-3)!$
- $= 7! / 3!4! = 7\times6\times5\times4! / (3\times2\times4!) = 7\times6\times5 / (3\times2) = 35$ ways
- Hence, 35 selections (groups) will be there such that all three balls are red.
- (b) None of the three will be red if these are chosen from (6 white and 4 blue balls) 10 balls and this can be done in
- $10\ C3 = 10!/\{3!(10-3)!\} = 10! / 3!7!$
- $= 10\times9\times8\times7! / (3\times2\times1\times7!) = 10\times9\times8 /(3\times2) = 120$ ways
- (c) Hence, by generalized fundamental principle, the number of groups of three balls such that one is of each colour = $7\times6\times4 = 168$ ways.

Properties of nCr :

- ${}^{n}C_r = {}^{n}C_{n-r}$
- ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$
- **${}^{n}C_o$** $= n!/\{0!\,(n-0)!\} = n!\,/\,n! =$ **1.**
- **${}^{n}C_n$** $= n!/\{n!\,(n-n)!\} = n!\,/\,n!\,.\,0! =$ **1**

- **Example 14: Pg 5.20**
- Find x if 12C5 +2 12C4 +12C3 = 14Cx
- **Solution:** L.H.S = 12C5+ 2 12C4 + 12C3
- = 12C5+ 12C4 + 12C4 + 12C3
- = 13C5 + 13C4
- = 14 C5
- Also nCr = nCn–r. Therefore 14C5 = 14C 14–5 = 14C9
- Hence, L.H.S = 14C5 = 14C9 = 14Cx = R.H.S by the given equality This implies, either x = 5 or x = 9

- **Combinations of n different things taking some or all of n things at a time**
- **Result :** The total number of ways in which it is possible to form groups by taking some or all of n things $(2^n - 1)$.

- **Combinations of n things taken some or all at a time when n1 of the things are alike of one kind, n2 of the things are alike of another kind n3 of the things are alike of a third kind. etc.**
- **Result :** The total, number of ways in which it is possible to make groups by taking some or all out of n (=n1 + n2 + n3 +...) things, where n1 things are alike of one kind and so on, is given by
- { (n1 + 1) (n2 + 1) (n3 + 1)...} –1

Pg 5.24

- **Example 1:** How many different permutations are possible from the letters of the word `CALCULUS'?
- **Solution:** The word `CALCULUS' consists of 8 letters of which 2 are C and 2 are L, 2 are U and the rest are A and S. Hence , by result (I), the number of different permutations from the letters of the word `CALCULUS' taken all at a time
- = 8! 2!2!2!1!1!
- 8 ´ 7 ´ 6´ 5 ´ 4 ´ 3 ´ 2
- = 2 ´ 2 ´ 2
- = 7 × 6 × 5 × 4 × 3 × 2 = 5,040

Pg 5.24

- **Example 2:** In how many ways can 17 billiard balls be arranged , if 7 of them are black, 6 red and 4 white?
- **Solution:** We have, the required number of different arrangements:
- 17! = 40,84,080
- 7! 6! 4!

- **Example 4:** A man has 5 friends. In how many ways can he invite one or more of his friends to dinner?
- **Solution:** By result, (III) of this section, as he has to select one or more of his 5 friends, he can do so in 25 – 1 = 31 ways.
- **Note :** This can also be done in the way, outlines below. He can invite his friends one by one, in twos, in threes, etc. and hence the number of ways.
- = 5C1+ 5C2 +5C3 +5C4 +5C5
- = 5 + 10 +10 + 5 + 1= 31 ways.

- **Example 6:** Find the number of ways of selecting 4 letters from the word `EXAMINATION'.
- **Solution:** There are 11 letters in the word of which A, I, N are repeated twice. Thus we have 11 letters of 8 different kinds (A, A), (I, I), (N, N), E, X, M, T, O. The group of four selected letters may take any of the following forms:
- Two alike and other two alike
- Two alike and other two different
- All four different
- In case (i), the number of ways = 3C2 = 3.
- In case (ii), the number of ways = 3C1 × 7C2 = 3 × 21 = 63.
- 8 ´ 7 ´ 6 ´ 5
- In case (iii), the number of ways = 8C4 =
- 1 ´ 2 ´ 3 ´ 4
- = 70
- Hence , the required number of ways = 3 + 63 + 70 = 136 ways

Ex 5C – No 8

- Out of 7 gents and 4 ladies a committee of 5 is to be formed. The number of committees such that each committee includes at least one lady is
- Possible combinations
- = 1L 4G
- 2L 3G
- 3L 2G
- 4 L 1 G
- = 35*4 + 35*6 + 4.21 + 1.7 = 441

No 12

- The number of straight lines obtained by joining 16 points on a plane, no three of them being on the same line is
  - *120 (b) 110 (c) 210 (d) none of these*
- 16C2= 120

No 14

Every two persons shakes hands with each other in a party and the total number of hand

- shakes is 66. The number of guests in the party is
- (a) 11 (b) 12 (c) 13 (d) 14

- Here Nc2= 66
- So n = 12

No 18

- 8 points are marked on the circumference of a circle. The number of chords obtained by joining these in pairs is
  - *25 (b) 27 (c) 28 (d) none of these*
  - *Ans = 8C2 = 28*

Recap

1. Difference of P& C
2. Standard Results
3. Problem solution

THANK YOU

